Using Non-linear Least Squares for Minimizing the Reprojection Error in Image Stitching

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Overview

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   - Drone Images: The More General Case
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   - Use Non-linear Least Squares to Minimize Reprojection Error
Background and Motivation
The Gantry and Large Scale Mosaicking

World’s Largest High-throughput Phenotyping Machine

The video
The Gantry and Large Scale Mosaicking

- Phenotype: the set of observable characteristics of an individual resulting from the interaction of its genotype with the environment.
- Genotype: the genetic constitution of an individual organism.
- The Goal of the Project is to find genes that makes individual plants to have a specific characteristics, i.e. Drought Resistant.
The Gantry and Large Scale Mosaicking

The Gantry:

- 2 Acres of land
- Half a dozen sensors
  - RGB cameras (0.3 mm per pixel)
  - PS II for Photosynthesis
  - FLIR / Thermal Camera
  - Hyper Spectral Camera
  - 3D Laser Scanner
The Gantry and Large Scale Mosaicking

- Gantry moves N to S and E to W
- Heavy metals and magnetic fields on the machine
- Relative Coordinate System: Barcodes on rails
- Conversion to GPS is noisy
- Wind causes displacement of the camera box

Resulting GPS is not enough for stitching images.

- Estimate pairwise transformations (translation is enough in many cases)
- Correct GPS coordinates (geo-correction)
Drone Images: The More General Case

- Drone is used more often
- More general and distinct features
- Challenge: transformation is not only *translation*.
  - Rotation
  - Scale
  - Tilting
Consistency: Important in Large Scale Mosaicking

- Pairwise image stitching has beaten to death
- Multi image stitching is more interesting
- Challenges
  - Minor error can propagate throughout the mosaic
  - Drift and inconsistency
- Old approach for multi image stitching in the literature
  - Iterative stitching: drift can cause huge inconsistency
Better approach

- Optimize pairwise transformations to minimize a score (projection error)
- Non-linear and slow procedure
My Goal: Linear Method for Minimizing the Drift Error

- **My goals**
  - Control the drift
  - Speed up the process
- **Proposed method works perfectly for the gantry**
- **Make it a general method for other applications**

**This presentation**
- Discuss one of the methods in the literature
- Illustrate the challenges of the mostly used approach
Image Stitching Short Review

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Image Stitching Short Review

Pairwise Transformations

MGRAPH Method for Large Scale Image Stitching

Non-linear Least Squares Optimization

Use Non-linear Least Squares to Minimize Reprojection Error
Pairwise Transformations between Pairs of Images

Four famous geometric transformations:

- Translation
- Similarity
- Affine
- Homography
Pairwise Transformations between Pairs of Images

Translation
Pairwise Transformations between Pairs of Images

Similarity
Pairwise Transformations between Pairs of Images

Affine and Homography
Estimating Pairwise Transformation using SIFT

Key point Matching

- **Stitch two images**
- Find important visual keypoints in each image (SIFT/SURF)
- Match the keypoints based on their descriptor (KNN)
- Estimate the desired transformation (RANSAC)
Estimating Pairwise Transformation using SIFT
Key point Matching
MGRAPH Method for Large Scale Image Stitching
Creating a GRAPH of Image Adjacency

- Works on Drone images (swarm of drones)
- Estimate pairwise transformation for each image and its neighbor
- Form a Graph that represents the adjacencies between images (build it on the go)
- Merge and split the graph as needed (weak and strong edges)
- Extract a minimum spanning tree from this graph
- Calculate absolute homographies w.r.t the root (reference)
- Minimize the reprojection error
- Use absolute homographies to warp and stitch the images
Reference Points and Absolute Homographies

- Matches between image i and j
  \[ M_{ij} = \langle p_i, p_j \rangle \]

- Pairwise Homography (similarity in their case)
  \[ p_i \approx H_{ij} p_j \]

- Absolute Homographies
  \[ H_{iR} = H_{ij} H_{jR} \]

  Takes a point from coordinate system of R to the coordinate system of i
Reprojection Error using Absolute Homographies

- Projection error

\[ r = p_i - H^{ij} p_j \]

- Using the underlying graph, the absolute homographies and SIFT matching form the following optimization function (reprojection error)

\[ \sum_i \sum_j \sum_{p \in M_{ij}} d(p_i, H^{iR}(H^{jR})^{-1} p_j) \]
Non-linear Least Squares Optimization

- Minimize the reprojection error for parameters of absolute homographies

\[ \sum \sum \sum_{p \in M_{ij}} d(p_i, H^R_i (H^R_j)^{-1} p_j) \]

- Because of the multiplication of the two H and the inverse this method is a Non-linear least squares (the function d).
Non-linear Least Squares Optimization

- Non-linear least squares:

\[ \arg \min_{\theta} S = \sum_{i=1}^{m} r_i^2 \]

\[ r_i = y_i - f(x_i, \theta) \]

- In our case:
  - \( p_i \)s and \( p_j \)s are in accordance \( x_i \)s and \( y_i \)s.
  - \( H_{iR} \)s are in accordance to \( \theta \) and \( f \) is \( H_{iR} (H_{jR})^{-1} p_j \).
  - \( d \) is the squared of the residual or the euclidean distance.
Non-linear Least Squares Optimization

- Non-linear Least Squares

\[ \arg \min_{\theta} S = \sum_{i=1}^{m} (y_i - f(x_i, \theta))^2 \]

- Available methods to optimize
  - GaussNewton method
  - LevenbergMarquardt algorithm
  - QR decomposition
  - Singular Value Decomposition
  - Gradient Methods
Use Non-linear Least Squares to Minimize Reprojection Error

- Two libraries for Non-Linear Least Squares optimizer
  - Python Scipy Non-Linear Least Squares optimizer
    - Uses QR method
    - Form Jacobian and decompose it to QR.
    - Form system of equations and solve for parameters
  - C++ Ceres
    - Uses Levenberg-Marquardt algorithm
    - Remember LMA interpolates between the Gauss-Newton algorithm gradient descent method.
    - Calling a C++ function from python (wrapper for Ceres) not successful so far.
- Non-linear optimization is usually slow and inaccurate in some cases.
Results

MGRAPH
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Use Non-linear Least Squares to Minimize Reprojection Error
Results
Results

Gantry Ortho
Thank you Very much for you attention.

I will upload the slide to my homepage at http://vision.cs.arizona.edu/ariyanzarei/